Primordial non-Gaussianity from two curvaton decays

Hooshyar Assadullahi (ICG. Portsmouth)

Phys. Rev. D 76,103003(arxiv:0708.0223v1)
H. Assadullahi, J. Valiviita, D. Wands

1th of July-Sheffield university
Outline:

• Primordial curvature perturbation.
• Curvaton scenario.
• Non-Gausianity with one curvaton.
• Non-Gausianity with two curvatons.
• Conclusion.
Curvature perturbation (first order):

• The gauge invariant, used to describe the large-scale curvature perturbation is:

\[ \zeta \equiv -\psi - H \frac{\delta \rho}{\dot{\rho}} \]

• It is curvature perturbation on uniform density gauge, and the density perturbation on uniform curvature slices.

Generally curvature perturbation is constant if:

1. Consider large scale
2. Pressure is only function of density (Baryotropic)
3. Non-interacting system
Single field inflation:

- In inflation models, the inflaton field plays two roles: makes the universe expand enough and generates primordial density perturbation.

- If the inflaton made primordial density perturbations, introduce extra fine tuning into the model: for example, inflaton mass could be large \( (m \approx H) \), without considering perturbations.

Curvaton scenario:


perturbations in some field other than the inflaton could be responsible for the primordial density perturbation \( \Rightarrow \) curvaton
Generating curvature perturbation by curvaton:

- The curvaton field is supposed to be light during inflation, with small effective mass, $m_\sigma \ll H$

- At end of inflation the energy of the inflaton field is converted into radiation $\rho_r \propto a^{-4}$

- After inflation $H$ is a decreasing function of time. Finally $m_\sigma^2 > H^2$ and curvaton starts to oscillate.

- The curvaton field behaves like pressure less matter $\rho_\sigma \propto a^{-3}$

- so $\frac{\rho_\sigma}{\rho_r} \propto a$

- Decay rate ($\Gamma$) of curvaton is negligible as long as $H > \Gamma$

- Curvaton decays some time before nucleosynthesis. $H_{\text{dec}} = \Gamma$
Curvaton perturbations:

- The curvaton field is supposed to be practically free during inflation, so it’s power spectrum at Hubble exit is:
  \[
  \frac{1}{P_{\delta\sigma}(k)} \approx \frac{H^*}{2\pi}
  \]

- The energy density in oscillating field after inflation is:
  \[
  \rho_\sigma(x,t) = m_\sigma^2 \bar{\sigma}^2(x,t)
  \]

- The perturbation of the curvaton field is:
  \[
  \zeta_\sigma = \frac{1}{3} \frac{\delta \rho}{\rho_\sigma} \approx \frac{2}{3} \frac{\delta \sigma}{\sigma}
  \]
• Considering two fields (radiation and curvaton) the total curvature perturbation will be:

\[ \zeta_{tot} = \frac{\dot{\rho}_r}{\dot{\rho}} \zeta_r + \frac{\dot{\rho}_\sigma}{\dot{\rho}} \zeta_\sigma \]

• Curvaton scenario corresponds to the case where the density perturbation in the radiation produced at the end of inflation is negligible:

\[ \zeta_r \approx 0 \]

Before curvaton decay
\[ \Rightarrow \zeta_{tot} = \frac{-3H(\rho_\sigma + p_\sigma)}{-3H(\rho_\sigma + p_\sigma) - 3H(\rho_r + p_r)} \zeta_\sigma = \frac{3\rho_\sigma}{3\rho_\sigma + 4\rho_r} \zeta_\sigma \]

After curvaton decay
\[ \Rightarrow \zeta_{tot} = r \zeta_\sigma \quad r = \left( \frac{3\rho_\sigma}{3\rho_\sigma + 4\rho_r} \right)_{\text{decay}} \]

• If the curvaton completely dominates the energy density before it decays we have \( r = 1 \), otherwise \( r < 1 \).
Non-Gaussianity:

- The simplest (possible) non-Gaussianity is caused by the square of the first order perturbation.
- Up to second order the perturbation can be written:

\[ \zeta = \zeta_1 + \frac{3}{5} f_{NL} \zeta_1^2 \]

- The primordial power spectrum:

\[ \left\langle \zeta_{(k_1)} \zeta_{(k_2)} \right\rangle = (2\pi)^3 P_\zeta(k_1) \delta^3(k_1 + k_2) \]

- The primordial bispectrum is:

\[ \left\langle \zeta_{(k_1)} \zeta_{(k_2)} \zeta_{(k_3)} \right\rangle = (2\pi)^3 B(k_1 + k_2 + k_3) \delta^3(k_1 + k_2 + k_3) \]

- The bispectrum vanishes for a purely Gaussian distribution
- The bispectrum relative to the power spectrum is commonly parameterized in terms of \( f_{NL} \):

\[ B(k_1 + k_2 + k_3) = \frac{6}{5} f_{NL} [P_{(k_1)} P_{(k_2)} + 2 \text{ perm}] \]
Non-Gausianity with one curvaton (M. Sasaki, J. Valiviita & D. Wands (Phys. Rev. D 74, 103003))

• In the sudden decay approximation, the bi-spectrum calculation leads to:

\[ f_{NL} (r) = \frac{5}{4} \frac{1}{r} (1 + \frac{gg''}{g'^2}) - \frac{5}{3} - \frac{5}{6} r \]

Where \( g \) describes the evolution of the curvaton between Hubble-exit during inflation and the beginning of the field oscillations:

\[ \sigma_{osc} = g \sigma (\sigma_*) \]

• If we consider \( r << 1 \) and linear evolution \( (g'' = 0) \):

\[ f_{NL} = \frac{5}{4} \frac{1}{r} \]

• The non-linearity becomes large only if the transfer of curvaton to curvature perturbation is inefficient (curvaton decays before it dominates the universe).
Non-Gausianity with two curvatons: (H. Assadullahi, J. Valiviita, D. Wands Phys. Rev. D 76, 103003)

- We assume that the curvaton $a$ decays first when $H = \Gamma_a$ followed by the decay of the curvaton $b$ when $H = \Gamma_b$, ($\Gamma_b < \Gamma_a$).
- The primordial curvature perturbation after both curvatons decay can be written up to second order in terms of the first order curvaton perturbations:

$$\zeta_2 = \zeta_{2(1)} + \frac{1}{2} \zeta_{2(2)} = A \zeta_{a(1)} + B \zeta_{b(1)} + \frac{1}{2} C \zeta_{a(1)}^2 + \frac{1}{2} D \zeta_{b(1)}^2 + \frac{1}{2} E \zeta_{a(1)} \zeta_{b(1)}$$

- We also find the relation between power spectra of each curvaton field and power spectrum of curvature perturbation:

$$P_{\zeta_b} = \frac{\beta^2}{A^2 + \beta^2 B^2} P_{\zeta} \quad P_{\zeta_a} = \frac{1}{A^2 + \beta^2 B^2} P_{\zeta}$$

Where $\beta$ is the ratio between power spectrum of $\zeta_b$ and $\zeta_a$:

$$P_{\zeta_b} = \beta^2 P_{\zeta_a}$$

- If we use all above relations we get:

$$\left\langle \zeta(k_1) \zeta(k_2) \zeta(k_3) \right\rangle = \frac{A^2 C + \beta^4 B^2 D + \frac{1}{2} \beta^2 ABE}{(A^2 + \beta^2 B^2)^2} \left\{ P_{\zeta}(k_1) P_{\zeta}(k_2) \right\} (2\pi)^3 \delta^{(3)}(k_1 + k_2 + k_3) + \text{perms}$$
So we can derive $f_{NL}$ for two curvatons:

\[
\frac{f_{NL}}{} = \frac{5}{6} \frac{CA^2 + \frac{1}{2} \beta^2 EAB + \beta^4 DB^2}{(A^2 + \beta^2 B^2)^2}
\]

- At first order we have:

\[
\zeta = r_a \zeta_{a,\text{in}} + r_b \zeta_{b,\text{in}}
\]

\[
r_a = [(1 - f_{b2})(3 + f_{a1})f_{a1}]/[3(1 - f_{b1}) + f_{a1}]
\]

\[
r_b = [(1 - f_{b1})f_{b2}(3 + f_{a1}) + f_{a1}f_{b1}]/[3(1 - f_{b1}) + f_{a1}]
\]

Where:

\[
f_{a1} = 3 \Omega_a / (4 \Omega_r + 3 \Omega_a + 3 \Omega_b) \text{ at first decay}
\]

\[
f_{b1} = 3 \Omega_b / (4 \Omega_r + 3 \Omega_a + 3 \Omega_b) \text{ at first decay}
\]

\[
f_{b2} = 3 \Omega_b / (4 \Omega_r + 3 \Omega_b) \text{ at second decay}
\]

- We can find A, B, C, D from second order calculation which are complicated functions of energy density of curvatons at their decay time.
$f_{NL}$ in various limits:

I: Single curvaton limits:

• If density of first or second curvaton is always zero ($f_a = 0$ or $f_{b_1} = f_{b_2} = 0$):

$$f_{NL} = \frac{5}{4r} - \frac{5}{3} - \frac{5r}{6} = f_{NL}^{\text{single}}$$

II: Simultaneous decay of two curvatons:

• If second curvaton decays at the same time as the first curvaton, we get:

$$\Omega_{b_1} = \Omega_{b_2} \Rightarrow f_{b_1} = (1 + f_{a_1}/3) f_{b_2}$$

• In the limiting case where the second curvaton is homogeneous ($\beta = 0$):

$$f_{NL} = f_{NL}^{\text{single}} (f_{a_1}) - \frac{5}{6} f_{b_1}$$

• When the first curvaton is homogeneous ($\beta \to \infty$) we get:

$$f_{NL} = f_{NL}^{\text{single}} (f_{b_1}) - \frac{5}{6} f_{a_1}$$
• In either case the example of two curvatons which decay at the same time does not reduce exactly to the case of a single curvaton.

• The minimum value for \( f_{NL} \) (for any value of \( \beta \)) is still that found for a single curvaton:

\[
\min( f_{NL} ) = -\frac{5}{4} = \min( f_{NL}^{\text{(Single)}} )
\]

III: Both curvatons subdominate at decay:

• If both curvatons decay before they dominate the energy density \( (f_{a_1}, f_{b_1}, f_{b_2} \ll 1) \) we can simplify the non-Gaussianity parameter:

\[
f_{NL} = \frac{5}{4} \frac{r_a^3 + \beta^4 r_b^3}{(r_a^2 + \beta^2 r_b^2)^2}
\]

• Like for one curvaton, \( f_{NL} \) is big if both curvatons are subdominant when they decay.

• But if the last curvaton is homogenous \( (\beta = 0) \), it dilutes the effect of first curvaton and we can have big non-Gaussianity even when both curvatons dominate before they decay. (this is a novel effect that has not been discovered with one curvaton)

\[
f_{NL} \propto \frac{1}{r_a}, \quad r_a \propto f(a_1)[1 - f(b_2)] \Rightarrow (if) \quad f(a_1), \quad f(b_2) \equiv 1 \Rightarrow f_{NL} \gg 1
\]
General case:

\[ f_{NL} \text{ when } P_{\zeta_a} = P_{\zeta_b} \ (\beta = 1) \]
Conclusion:

• The curvaton is the proposal for how primordial density perturbation may arise from vacuum fluctuations of a light scalar field that does not dominate during inflation.

• Detection of primordial non-Gaussianity could provide support for a curvaton-type scenario.

• If we take seriously the multiplicity of scalar fields in the early universe, we should consider more than one field can contribute to the primordial density perturbation.

• With one or two curvatons fields the non-Gaussianity becomes large if the transfer of curvaton to curvature perturbation is inefficient.

• With two curvaton we can have big non-Gaussianity even when the two fields dominate before they decay, if the second curvaton is homogeneous. In this case the second curvaton dilutes the first order perturbation.

• It should be straightforward to extend our analysis to three or more curvatons.
\[ f_{NL} \quad \text{When } \beta = 0 \]